Problem 2: Bathtub Curtain Rod

(a)

Maximum Shear Load

Under normal operating conditions, it is reasonable to assume that the worst load experienced by a shower curtain rod would be applied by either a particularly heavy shower caddy or wet towels. Many people have hanging organizers in their showers, and while they are usually attached to shower heads, it's not unreasonable to think that a user may hang theirs from a shower rod instead. Similarly, we can expect people to toss towels over the rod after getting out of the shower, which can be heavy, especially when wet. Moreover, barring extreme circumstances (like someone hanging from or pulling hard on the rod), there are few standard items in the bathroom that weigh as much as these items.

Assuming a few large (say, 32 fl oz) bottles of shampoo/conditioner/body wash, plus other miscellaneous items (bar soap, washcloth, etc) and the weight of a caddy itself, a reasonable upper limit for the weight of a shower organizer falls around 20lbf. This is likely more on the conservative side, but if a user has several large bottles of product, or a metal caddy, it's possible that the estimate would be met. Additionally, the upper bound for a wet towel would fall just short of 4 lbf.

In this analysis, it also seems reasonable to consider the weight of the shower curtain itself. While the curtain is likely quite light, some users clip small weights to their curtains to prevent them from blowing in. In this case, a reasonable upper bound would also be 4lbf.

In total, under the assumption of the heaviest possible caddy, two towels, and the weight of the curtain applied to one spot, the worst load experienced the rod would come out to be 32lbf.

Worst Location of Application

When it comes to the placement of the caddy and towels, the direct center of the rod is the worst location based on shear and bending moment diagrams. As illustrated in figure 1, the shear force throughout the bar is either $+\frac{1}{2}P$ or $-\frac{1}{2}P$ (ie, +/- 16 lbf), since there are only three forces of concern acting on the rod. In turn, when this shear force is integrated, it's easy to see that the maximum bending moment is 480 in-lbf. Importantly, if the force were to be applied off-center, this same maximum value would not be reached. We can illustrate this fact using a simple analysis that is specific to this case. Using the notation from the figure and adding an ambiguous location of load application x_P, force and moment equilibrium equations will look like

$$\sum F_y = y_L + y_R - 32 = 0$$

$$\sum M_R = -60y_L + (60 - x_P)32 = -60y_L + 1920 - 32x_P$$

$$y_L = \frac{1}{60}(1920 - 32x_P)$$

Then, taking the maximum moment in the bar to be at the point of the applied load (based on the logic that we integrate M(x) to yield V(x)), we get

$$M_{max} = \frac{x_P}{60}(1920 - 32x_P) = 32x_P - \frac{32}{60}x_P^2$$

Setting the derivative of this expression to 0 to find the maximum value:

$$M'_{max} = 2\frac{32}{60}x_P - 32 = 0$$
$$x_P = 30$$

Therefore, the highest moment in the bar occurs when $x_P = 30$", which proves that applying the load in the center of the bar will produce the worst possible loading scenario.

This is also why the load is idealized as a point force even though such a thing is impossible—placing all the weight in the center will yield the most extreme force values. This makes intuitive sense, too, since loads applied furthest from each point of attachment will yield the worst case scenario.

(b)

Material Selection

While there are a wide range of prices for shower rods, the simple ones seem to sell for an average of around $20. Consistent with this value, a reasonable material is 304 grade stainless steel. This grade is the most widely-used across a variety of applications due to its good corrosion resistance and strength, as well as its accessibility and low cost. It is an alloy composed of about 16 and 24 percent chromium and up to 35 percent nickel, as well as small amounts of carbon and manganese. For the shower rod—an everyday household item that isn't subjected to any extreme conditions—304 grade therefore seems perfectly suitable.

Under ambient operating conditions in a bathroom during a shower (ie elevated humidity and room temperature), 304 grade stainless steel will undergo ductile failure. In particular, 304 grade is classified within the "Austenitic" family of stainless steel, which is one of the most ductile groups. The slightly elevated temperature and humidity will therefore have negligible effect on the type of failure the rod undergoes.

Safety Factor Selection

Because of the material's widespread use and reliable properties, a safety factor of 1.5 then seems appropriate for use in creating the design. Aside from the alloy being very reliable, the conditions to which a shower rod will be subjected to are also controllable, and the loads are well-defined. Shower rods are used indoors in relatively invariable environments, and the loads as discussed in part (a) are conservative. There are only so many forces that can be applied to a shower rod, and there is a reasonable expectation that consumers will not attempt to apply extreme loads.

Additionally, there are no significant impact forces to consider, and the consequences to human safety are relatively low. If the shower rod were to fall, it may strike someone, but there is no significant risk of anything but minor injury. In contrast, increasing safety factor can rapidly increase price, and in a market where prices must be competitive, it does not make sense to overdesign beyond a reasonable limit.

(c)

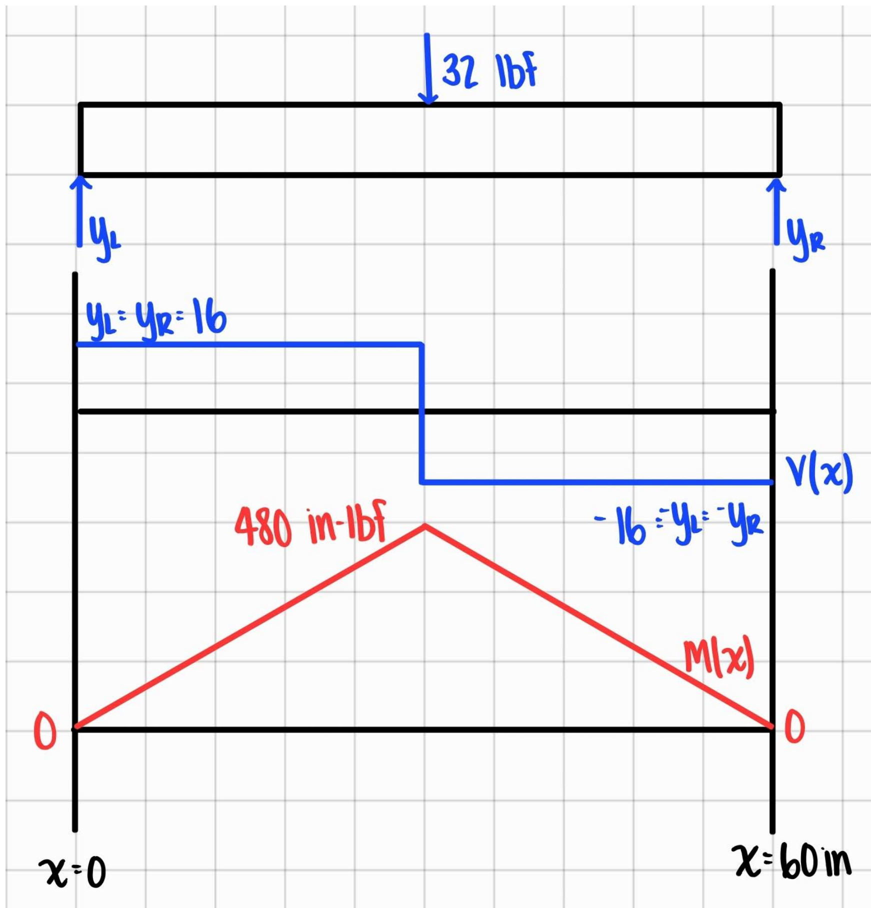

Figure 1: Shear and bending moments for the rod with 32lbf applied at the direct center (ie 30in). This is the worst case scenario for the location of the applied load.

To determine an appropriate r_0 for the bar, beam theory equations can be applied for the worst possible scenario. In this case, we will assume that the shower rod is simply supported at both ends, with the 32 lbf force discussed above acting as a point force at the center of the bar. While many shower rods might be attached more robustly to the wall (ie, anchored completely), using simple supports will produce more extreme forces throughout the bar, and is therefore a more conservative choice for this analysis. Additionally, many shower rods are not anchored completely to the wall, so this choice provides a more practical simulation. This setup is modeled in figure 1, and the equations for V(x) and M(x) are worked out below.

Determining reaction forces by symmetry:

$$\sum F_y = y_L + y_R - P = 0$$
$$\implies y_L = y_R = 16 \text{ lbf}$$

There are no reaction moments because of the type of support selected. Then, explicitly, the shear and bending moment equations are:

$$V(x) = \begin{cases} 16 & x < 30 \\ -16 & x > 30 \end{cases} \tag{1}$$

$$M(x) = \begin{cases} 16x & x < 30 \\ -16x & x > 30 \end{cases} \tag{2}$$

Next, to determine r_0, we first need to determine the maximum value of σ_{xx} (ie, σ_{max}). To do this, both the bending moment M and the radial displacement y need to be maximized. This corresponds to the point in the center of the bar lengthwise, where M reaches its maximum of 240 in-lbf, as well as the largest radial point, which in this case will be $-r_0$. (r_0 would also work, but using $-r_0$ cleans up signs and keeps the problem simple.) Using these values:

$$\sigma_{xx} = -\frac{My}{I_{xx}} = -\frac{My}{\frac{\pi r_0^4}{4}} = -\frac{4My}{\pi r_0^4}$$

$$\implies \sigma_{max} = -\frac{4(480)(-r_0)}{\pi r_0^4} = \frac{1920}{\pi r_0^3}$$

For the 304 grade stainless steel, Solidworks gives a yield strength of 30000 psi, which will be used as the upper limit of what the material can bear. Finally, a factor of safety of 1.5 will be applied to the maximum load consistent with the reasoning above. Setting σ_{max} equal to the yield strength given by Solidworks:

$$30000 = \frac{1920}{\pi r_0^3}(1.5)$$

$$r_0^3 = 0.031$$

$$\boxed{r_0 = 0.31in}$$

Therefore, the bar would need to have a radius of 0.31 inches to satisfy all the criteria above, which seems reasonable given the typical size of shower rods on the market and the fact that this design is completely solid. Running this analysis in Solidworks also confirms a similar result, as indicated in the appendix.

(d)

Appropriate Hollow Rod Dimensions

304 Stainless Steel tubes are readily available on McMaster Carr with a wide variety of outer diameters and wall thicknesses. Starting with a 0.12" wall thickness and an outer radius of 0.3125" to be consistent with part (c), the same procedure from above yields

$$\sigma_{xx} = -\frac{My}{I_{xx}} = -\frac{4My}{\pi(r_o^4 - r_i^4)}$$

$$\sigma_{max} = -\frac{4(480)(-r_o)}{\pi(r_o^4 - r_i^4)} = \frac{1920r_o}{\pi(r_o^4 - r_i^4)}$$

$$30000 = \frac{1920(0.3125)}{\pi(0.3125^4 - 0.1925^4)}(SF)$$
$$SF = 1.28$$

Here, the factor of safety falls short of the desired value of 1.5 To increase it, moment of inertia I_{xx} must increase, so outer radius/wall thickness should increase correspondingly. Increasing outer radius to 0.375" (the next size avaliable on McMaster Carr) yields

$$30000 = \frac{1920(0.375)}{\pi(0.375^4 - 0.0.255^4)}(SF)$$
$$SF = 2.035$$

This is far too drastic of a jump, and the safety factor now indicates that the rod has been over-designed. To compensate, the thickness can be decreased to drive the safety factor back down towards 1.5. Changing thickness to 0.083" (another McMaster Carr option) gives

$$30000 = \frac{1920(0.375)}{\pi(0.375^4 - 0.292^4)}(SF)$$
$$SF = 1.64$$

While much closer to 1.5, this safety factor is still slightly too high, so we can decrease thickness again, this time to 0.065":

$$30000 = \frac{1920(0.3125)}{\pi(0.375^4 - 0.31^4)}(SF)$$
$$SF = 1.37$$

This safety factor is now too low, indicating that the previous solution is likely one of the best commercially available options. Of course, it's possible that diameter and thickness could be further manipulated to better approach a SF of 1.5, but using the second option as the solution seems completely reasonable in this case, since it has a safety factor just slightly over the target.

$$\boxed{r_o = 0.375\text{" and } h = 0.083\text{", which yields SF} = 1.64}$$

Again, all of these calculations can be verified by Solidworks analyses. Those results (and screenshots) are included in the appendix.

Mass Calculations

Then, using the the mass density of 0.2890 lb/in^3 given by Solidworks, the mass of both this hollow design and the solid one from part (c) can be determined. By coincidence, the diameter of the design in part (c) is $\frac{5}{8}$", which is also available on McMaster Carr for purchase. Results are summarized in the table below:

Type	**Mass**	**Safety Factor**	**Price**
Solid Rod, $r_0 = 0.31$"	5.32 lbs	1.5	\$30.52
Hollow Rod, $r_0 = 0.375$", $h = 0.083$"	2.45 lbs	1.64	\$40.44

The prices indicated above are obviously over the $20 estimate used at the beginning of the problem, but they are also priced for buying a single item. Buying in bulk like a manufacturer would do would likely decrease this unit price rather significantly.

(e)

Peak Normal Stress

Using the formula for σ_{max} derived above, the anticipated maximum normal stress for the design in part (d) is

$$\sigma_{xx} = -\frac{My}{I_{xx}} = -\frac{4(480)(0.375)}{\pi(0.375^4 - 0.292^4)} = 18327 \text{ psi}$$

Again, this occurs at the worst cross section, $x = 30$", and on the outer edge of the rod $y = 0.375$".

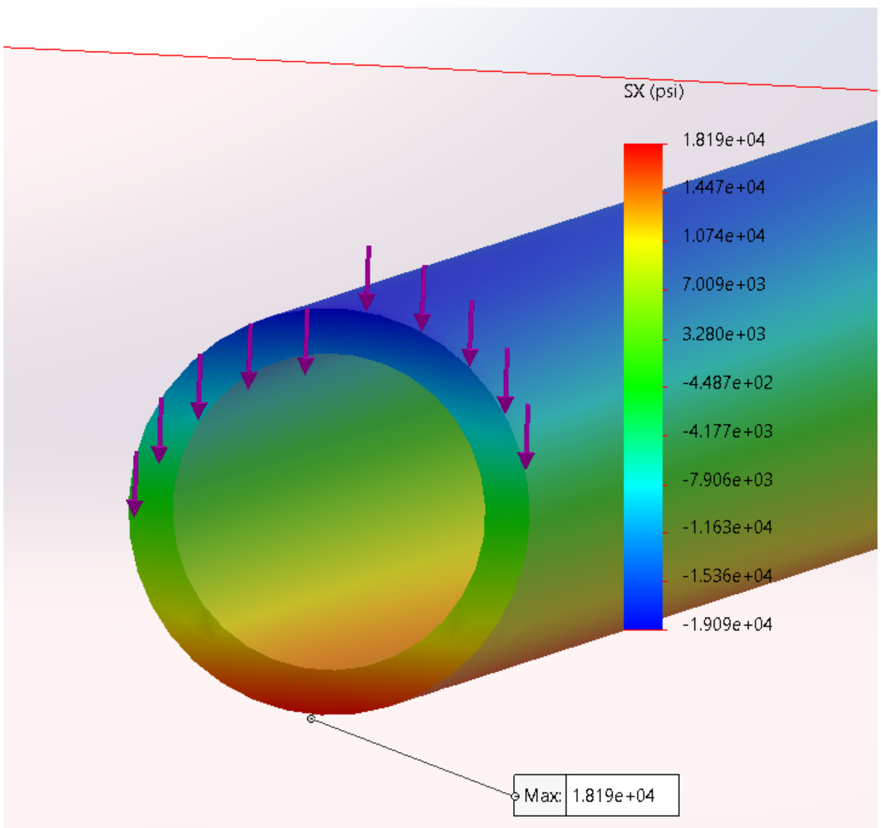

Figure 2: A cross section of the normal stress plot taken at $x = 30$". The maximum value of σ_{xx} is clearly labeled.

The same analysis in Solidworks produces a value of σ_{max} that agrees closely with the one predicted above, as indicated in figure 2. Namely,

$$\boxed{\sigma_{max} = 18190\text{psi}}$$

While not exactly identical, these two values have a percent error of only 0.75%, indicating that the model is a good representation of the calculations done by hand. Additionally, just as expected, the maximum stress occurs on the outside edge of the rod and at $x = 30$".

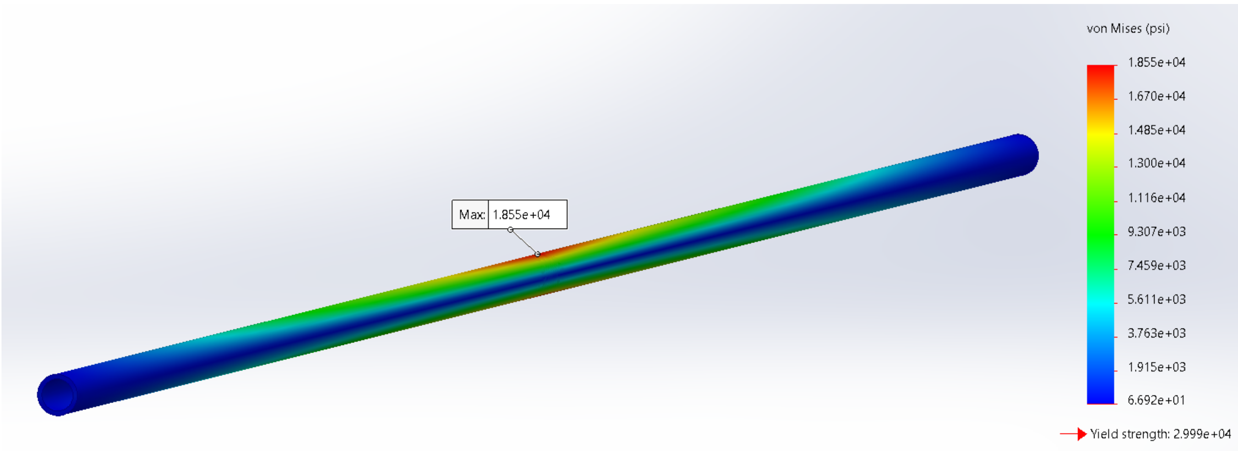

Figure 3: A plot of the von Mises stress throughout the bar. The maximum von Mises value is 18530 psi.

von Mises Stress Distribution

Finally, figure 3 shows the full von Mises criterion as a color map along the bar. This time, the worst value of stress is 18530 psi, which occurs at $x = 30$" on the top side of the bar. Using this value to determine how well the safety factor was met yields

$$SF = \frac{\text{yield strength}}{\text{maximum von Mises stress}} = \frac{30000}{18530} = 1.62$$

This safety factor is nearly identical to the one predicted by the earlier analysis, and therefore still meets the established criterion of 1.5. In this case, the Solidworks model has verified that the chosen dimensions satisfy the safety factor from part (b). As always, more information about this modeling is included in the appendix.

Appendix: Solidworks Simulation Results

For the simulations in this problem, the shower rod model was built according to the various physical dimension specifications, then loaded in the same way each time. Namely, each end of the bar was subject to a fixture that prevented movement in the positive y direction to simulate the simple supports used in the problem. Then, the point at the center of the bar lengthwise (x = 30") was loaded with a 32 lbf force in the negative y direciton. This load was applied across the top half of the circle defining the outer edge of the cross section of the bar. In the earlier definition of the point load, a 2D simplification was used, so extending the load in the z dimension makes sense in the 3D space. This load is illustrated in figure 4.

Additionally, when performing all of the below simulations, the finest mesh offered by Solidworks was used. Because the design has simple geometry with no sharp edges, the mesh converged quickly in every case. In other words, the coarser meshes produces more or less the same values as the finer ones. Because these simulations run so quickly, there was no reason to not use the finest possible mesh.

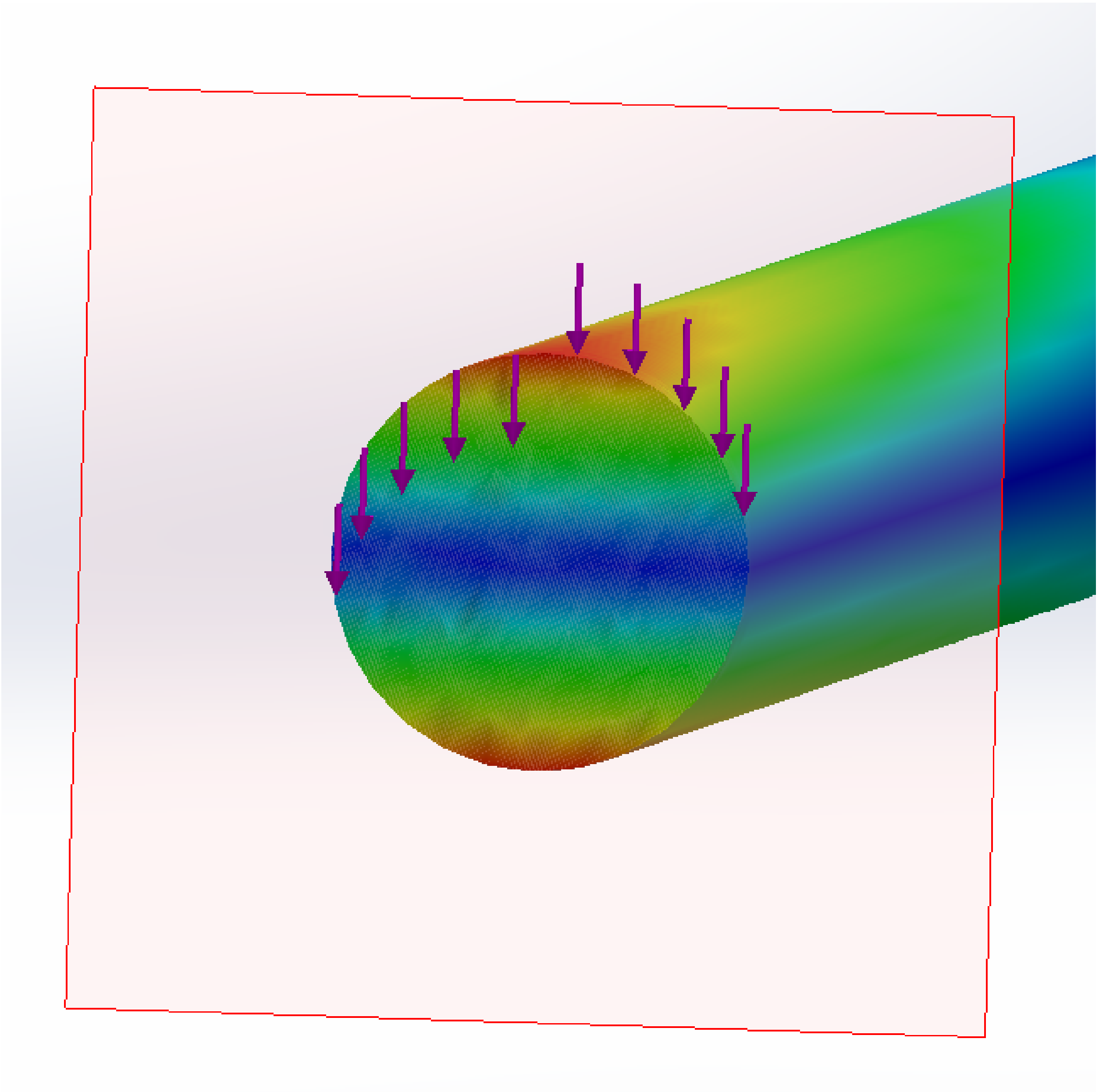

Figure 4: An illustration of how the center load was applied. This application attempts to act as a point force extended to 3D space.

The figures below show the verified Factor of Safety results for each trial performed in parts (c) and (d).

Clearly, there is close agreement between the simulation and the hand calculations each time.

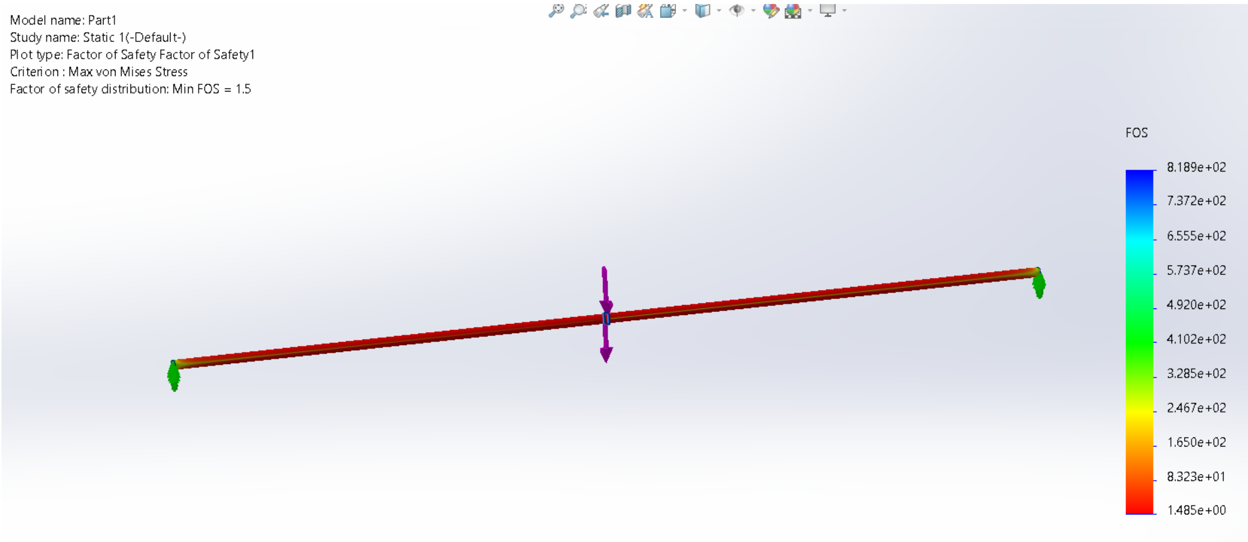

Figure 5: The solid bar trial with $r_0 = 0.31$" and resulting factor of safety of 1.5 (hand calculation) and 1.48 (Solidworks).

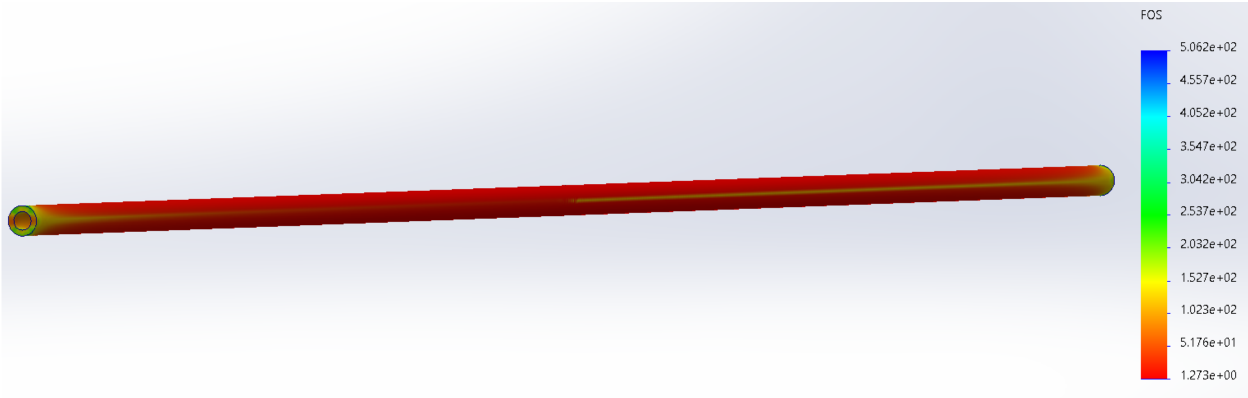

Figure 6: The hollow bar trial with $r_0 = 0.3125$", $h = 0.12$" and resulting factor of safety of 1.28 (hand calculation) and 1.273 (Solidworks).

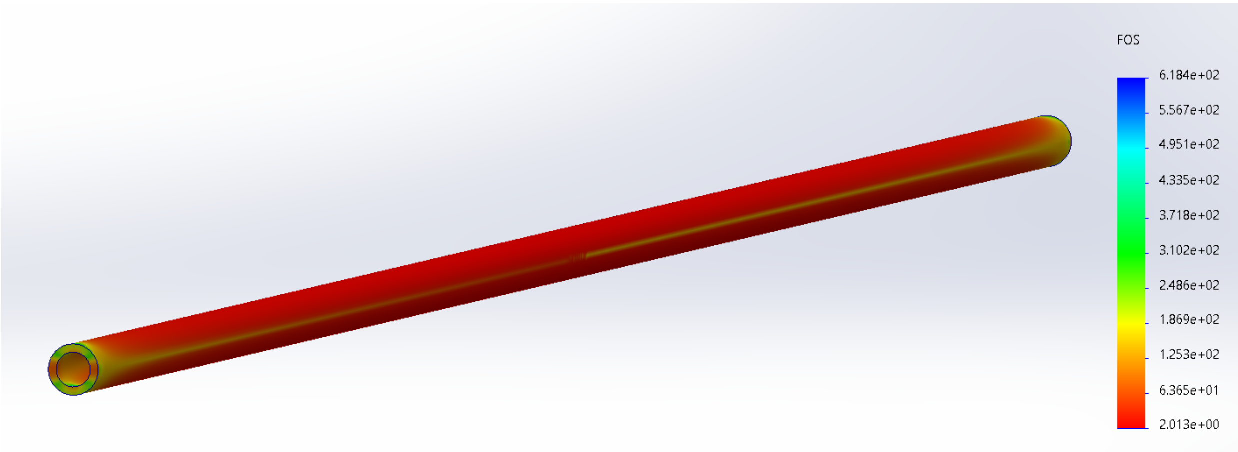

Figure 7: The hollow bar trial with $r_0 = 0.375$", $h = 0.12$" and resulting factor of safety of 2.035 (hand calculation) and 2.013 (Solidworks).

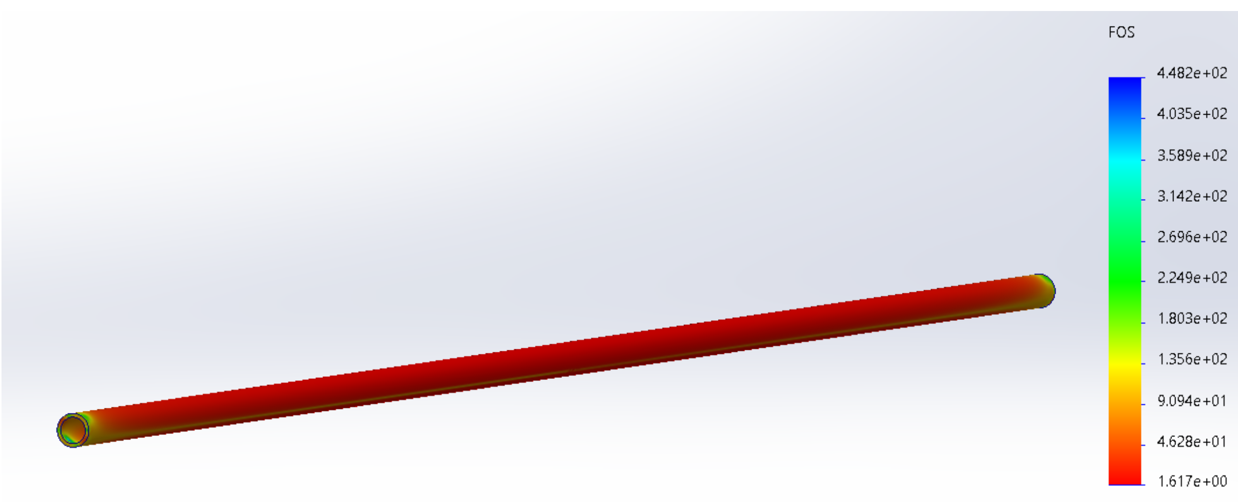

Figure 8: The hollow bar trial with $r_0 = 0.375$", $h = 0.083$" and resulting factor of safety of 1.64 (hand calculation) and 1.617 (Solidworks).